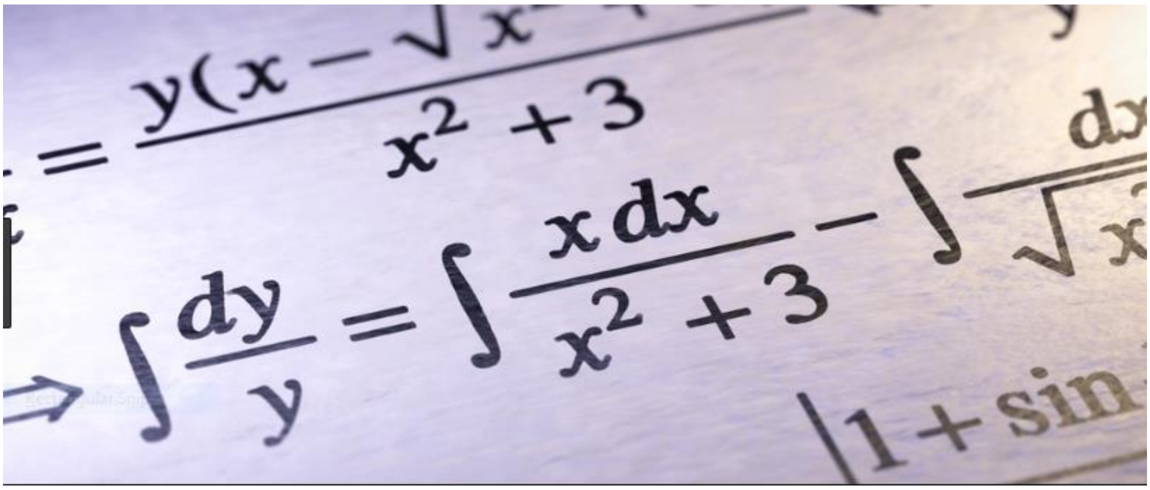
# Richard Challoner School

## MATHEMATICS DEPARTMENT

## Introduction to A level Maths

### INDUCTION BOOKLET



# INTRODUCTION TO A LEVEL MATHS AT RICHARD CHALLONER SCHOOL

Thank you for choosing to study Mathematics in the sixth form at Richard Challoner School. In year 12 you will study three elements - Pure Mathematics, Mechanics and Statistics. In order that you make the best possible start to the course, we have prepared this booklet for you.

It is vitally important that you spend time working through the questions in this booklet over the summer - you will need to have a good knowledge of these topics before you commence your course in September, and in order to **pass an algebra test during August**. You should have met all the topics before at GCSE. Work through the introduction to each chapter, making sure that you understand the examples. Then tackle the exercise – not necessarily every question, but enough to ensure you understand the topic thoroughly. The answers are given at the back of the booklet.

In addition, there are several websites where you will find helpful notes for each topic.

The first is www.mathcentre.ac.uk. On the left hand side click on "Topic", this will bring up a new page and you need to click on Algebra. This will bring you to a list of topics, you need to choose the topic(s) you are finding difficult. Once your topic has been selected you will then see a list of resources to use. The two most suitable ones are 'iPOD Video' and 'Teach Yourself'.

Others include www.vle.mathswatch.co.uk which you can use if you have a school login, and www.corbettmaths.com (see suggested videos numbers below) and www.mathsgenie.co.uk are both free.

In August you will sit a test to check how well you understand these topics, it is therefore important that you have answered the booklet before then. Your test result will indicate whether you are suitable to take A level maths and also if you will be required to attend additional support lessons over and above your timetabled ones.

We hope that you will use this introduction to give you a good start to your A level work and that it will help you enjoy and benefit from the course more.

Enjoy your summer break.

**Mr MacGreevy**

## CONTENTS

# Chapter 1: EXPANDING BRACKETS

To expand a single bracket, we multiply every term in the bracket by the number or the expression on the outside:

**Examples**

1) $3(x + 2y) = 3x + 6y$

2) $-2(2x - 3) = (-2)(2x) + (-2)(-3)$
$= -4x + 6$

To expand two brackets, we must multiply everything in the first bracket by everything in the second bracket. We can do this in a variety of ways, including
* the smiley face method
* FOIL (Fronts Outers Inners Lasts)
* using a grid.

**Examples**:

1) $(x + 1)(x + 2) = x(x + 2) + 1(x + 2)$

or

$(x + 1)(x + 2) = x^2 + 2 + 2x + x$
$= x^2 + 3x + 2$

or

| | $x$ | 1 |
|---|---|---|
| $x$ | $x^2$ | $x$ |
| 2 | $2x$ | 2 |

$(x + 1)(x + 2) = x^2 + 2x + x + 2$
$= x^2 + 3x + 2$

2) $(x - 2)(2x + 3) = x(2x + 3) - 2(2x + 3)$
$= 2x^2 + 3x - 4x - 6$
$= 2x^2 - x - 6$

or

$(x - 2)(2x + 3) = 2x^2 - 6 + 3x - 4x = 2x^2 - x - 6$

or

| | $x$ | -2 |
|---|---|---|
| $2x$ | $2x^2$ | $-4x$ |
| 3 | $3x$ | -6 |

$(2x + 3)(x - 2) = 2x^2 + 3x - 4x - 6$
$= 2x^2 - x - 6$

**EXERCISE A** Multiply out the following brackets and simplify.

1. $7(4x + 5)$
2. $-3(5x - 7)$
3. $5a - 4(3a - 1)$
4. $4y + y(2 + 3y)$
5. $-3x - (x + 4)$
6. $5(2x - 1) - (3x - 4)$
7. $(x + 2)(x + 3)$
8. $(t - 5)(t - 2)$
9. $(2x + 3y)(3x - 4y)$
10. $4(x - 2)(x + 3)$
11. $(2y - 1)(2y + 1)$
12. $(3 + 5x)(4 - x)$

## Two Special Cases

| | |
|---|---|
| **Perfect Square:** | **Difference of two squares:** |
| $(x + a)^2 = (x + a)(x + a) = x^2 + 2ax + a^2$ | $(x - a)(x + a) = x^2 - a^2$ |
| $(2x - 3)^2 = (2x - 3)(2x - 3) = 4x^2 - 12x + 9$ | $(x - 3)(x + 3) = x^2 - 3^2$ |
| | $= x^2 - 9$ |

**EXERCISE B** Expand and simplify:

1. $(x - 1)^2$
2. $(3x + 5)^2$
3. $(7x - 2)^2$
4. $(x + 2)(x - 2)$
5. $(3x + 1)(3x - 1)$
6. $(5y - 3)(5y + 3)$

# Chapter 2: SOLVING LINEAR EQUATIONS

When solving an equation, you must remember that whatever you do to one side must also be done to the other. You are therefore allowed to

- add the same amount to both side
- subtract the same amount from each side
- multiply the whole of each side by the same amount
- divide the whole of each side by the same amount.

If the equation has unknowns on both sides, you should collect all the letters onto the same side of the equation.

If the equation contains brackets, you should start by expanding the brackets.

A linear equation is an equation that contains numbers and terms in $x$.

**Example 1**: Solve the equation $64 - 3x = 25$

**Solution**: There are various ways to solve this equation. One approach is as follows:

Step 1: Add $3x$ to both sides (so that the $x$ term is positive): $64 = 3x + 25$

Step 2: Subtract 25 from both sides: $39 = 3x$

Step 3: Divide both sides by 3: $13 = x$

So the solution is $x = 13$.

**Example 2**: Solve the equation $6x + 7 = 5 - 2x$.

**Solution:**
Step 1: Begin by adding $2x$ to both sides $8x + 7 = 5$
(to ensure that the $x$ terms are together on the same side)

Step 2: Subtract 7 from each side: $8x = -2$

Step 3: Divide each side by 8: $x = -\frac{1}{4}$

**Exercise A**: Solve the following equations, showing each step in your working:

1) $2x + 5 = 19$ 2) $5x - 2 = 13$ 3) $11 - 4x = 5$

4) $5 - 7x = -9$ 5) $11 + 3x = 8 - 2x$ 6) $7x + 2 = 4x - 5$

**Example 3**: Solve the equation $2(3x - 2) = 20 - 3(x + 2)$

Step 1: Multiply out the brackets: $6x - 4 = 20 - 3x - 6$
(taking care of the negative signs)

Step 2: Simplify the right hand side: $6x - 4 = 14 - 3x$

Step 3: Add 3x to each side: $9x - 4 = 14$

Step 4: Add 4: $9x = 18$

Step 5: Divide by 9: $x = 2$

**Exercise B:** Solve the following equations.

1) $5(2x - 4) = 4$

2) $4(2 - x) = 3(x - 9)$

3) $8 - (x + 3) = 4$

4) $14 - 3(2x + 3) = 2$

**EQUATIONS CONTAINING FRACTIONS**

When an equation contains a fraction, the first step is usually to multiply through by the denominator of the fraction. This ensures that there are no fractions in the equation.

**Example 4**: Solve the equation $\frac{y}{2} + 5 = 11$

**Solution**:
Step 1: Multiply through by 2 (the denominator in the fraction): $y + 10 = 22$

Step 2: Subtract 10: $y = 12$

**Example 5**: Solve the equation $\frac{1}{3}(2x + 1) = 5$

**Solution**:
Step 1: Multiply by 3 (to remove the fraction) $2x + 1 = 15$

Step 2: Subtract 1 from each side $2x = 14$

Step 3: Divide by 2 $x = 7$

When an equation contains two fractions, you need to multiply by the lowest common denominator. This will then remove both fractions.

**Example 6**: Solve the equation $\frac{x+1}{4} + \frac{x+2}{5} = 2$

**Solution**:

Step 1: Find the lowest common denominator: The smallest number that both 4 and 5 divide into is 20.

Step 2: Multiply both sides by the lowest common denominator $\frac{20(x+1)}{4} + \frac{20(x+2)}{5} = 40$

Step 3: Simplify the left hand side: $\frac{\overset{5}{\cancel{20}}(x+1)}{\cancel{4}} + \frac{\overset{4}{\cancel{20}}(x+2)}{\cancel{5}} = 40$

$5(x + 1) + 4(x + 2) = 40$

Step 4: Multiply out the brackets: $5x + 5 + 4x + 8 = 40$

Step 5: Simplify the equation: $9x + 13 = 40$

Step 6: Subtract 13 $9x = 27$

Step 7: Divide by 9: $x = 3$

**Example 7**: Solve the equation $x + \frac{x-2}{4} = 2 - \frac{3-5x}{6}$

**Solution**: The lowest number that 4 and 6 go into is 12. So we multiply every term by 12:

$$12x + \frac{12(x-2)}{4} = 24 - \frac{12(3-5x)}{6}$$

| | |
|---|---|
| Simplify | $12x + 3(x-2) = 24 - 2(3-5x)$ |
| Expand brackets | $12x + 3x - 6 = 24 - 6 + 10x$ |
| Simplify | $15x - 6 = 18 + 10x$ |
| Subtract $10x$ | $5x - 6 = 18$ |
| Add 6 | $5x = 24$ |
| Divide by 5 | $x = 4.8$ |

**Exercise C**: Solve these equations

1) $\frac{1}{2}(x+3) = 5$

2) $\frac{2x}{3} - 1 = \frac{x}{3} + 4$

3) $\frac{y}{4} + 3 = 5 - \frac{y}{3}$

4) $\frac{x-2}{7} = 2 + \frac{3-x}{14}$

5) $\frac{7x-1}{2} = 13 - x$

6) $\frac{y-1}{2} + \frac{y+1}{3} = \frac{2y+5}{6}$

7) $2x + \frac{x-1}{2} = \frac{5x+3}{3}$

8) $2 - \frac{5}{x} = \frac{10}{x} - 1$

## FORMING EQUATIONS

**Example 8**: Find three consecutive numbers so that their sum is 96.

**Solution**: Let the first number be $n$, then the second is $n + 1$ and the third is $n + 2$.

Therefore $n + (n + 1) + (n + 2) = 96$

$$3n + 3 = 96$$
$$3n = 93$$
$$n = 31$$

So the numbers are 31, 32 and 33.

**Exercise D:**

1) Find 3 consecutive even numbers so that their sum is 108.

2) The perimeter of a rectangle is 79 cm. One side is three times the length of the other. Form an equation and hence find the length of each side.

3) Two girls have 72 photographs of celebrities between them. One gives 11 to the other and finds that she now has half the number her friend has.
Form an equation, letting $n$ be the number of photographs one girl had at the **beginning**.
Hence find how many each has **now**.

# Chapter 3: FACTORISING

## Common factors

We can factorise some expressions by taking out a common factor.

| | |
|---|---|
| **Example 1**: | Factorise $12x - 30$ |
| **Solution**: | 6 is a common factor to both 12 and 30. We can therefore factorise by taking 6 outside a bracket:<br>$12x - 30 = 6(2x - 5)$ |

| | |
|---|---|
| **Example 2**: | Factorise $6x^2 - 2xy$ |
| **Solution**: | 2 is a common factor to both 6 and 2. Both terms also contain an $x$.<br>So we factorise by taking $2x$ outside a bracket.<br>$6x^2 - 2xy = 2x(3x - y)$ |

| | |
|---|---|
| **Example 3**: | Factorise $9x^3y^2 - 18x^2y$ |
| **Solution**: | 9 is a common factor to both 9 and 18.<br>The highest power of $x$ that is present in both expressions is $x^2$.<br>There is also a $y$ present in both parts.<br>So we factorise by taking $9x^2y$ outside a bracket:<br>$9x^3y^2 - 18x^2y = 9x^2y(xy - 2)$ |

| | |
|---|---|
| **Example 4**: | Factorise $3x(2x - 1) - 4(2x - 1)$ |
| **Solution**: | There is a common bracket as a factor.<br>So we factorise by taking $(2x - 1)$ out as a factor.<br>The expression factorises to $(2x - 1)(3x - 4)$ |

## Exercise A

Factorise each of the following

1) $3x + xy$

2) $4x^2 - 2xy$

3) $pq^2 - p^2q$

4) $3pq - 9q^2$

5) $2x^3 - 6x^2$

6) $8a^5b^2 - 12a^3b^4$

7) $5y(y - 1) + 3(y - 1)$

## Factorising quadratics

**Simple quadratics: Factorising quadratics of the form** $x^2 + bx + c$

The method is:

Step 1: Form two brackets $(x \;\dots\;)(x \;\dots\;)$

Step 2: Find two numbers that multiply to give $c$ and add to make $b$. These two numbers get written at the other end of the brackets.

**Example 1**: Factorise $x^2 – 9x – 10$.

**Solution**: We need to find two numbers that multiply to make -10 and add to make -9. These numbers are -10 and 1.

Therefore $x^2 – 9x – 10 = (x – 10)(x + 1)$.

**General quadratics: Factorising quadratics of the form** $ax^2 + bx + c$

The method is:

Step 1: Find two numbers that multiply together to make $ac$ and add to make $b$.

Step 2: Split up the $bx$ term using the numbers found in step 1.

Step 3: Factorise the front and back pair of expressions as fully as possible.

Step 4: There should be a common bracket. Take this out as a common factor.

**Example 2**: Factorise $6x^2 + x – 12$.

**Solution**: We need to find two numbers that multiply to make $6 \times -12 = -72$ and add to make 1. These two numbers are -8 and 9.

Therefore, $6x^2 + x – 12 = \underbrace{6x^2 - 8x}\; + \underbrace{9x – 12}$

$= 2x(3x – 4) + 3(3x – 4)$ (the two brackets must be identical)

$= (3x – 4)(2x + 3)$

**Difference of two squares: Factorising quadratics of the form** $x^2 - a^2$

Remember that $x^2 - a^2 = (x + a)(x – a)$.

Therefore: $x^2 - 9 = x^2 - 3^2 = (x+3)(x-3)$

$16x^2 - 25 = (2x)^2 - 5^2 = (2x+5)(2x-5)$

Also notice that: $2x^2 - 8 = 2(x^2 - 4) = 2(x+4)(x-4)$

and $3x^3 - 48xy^2 = 3x(x^2 - 16y^2) = 3x(x+4y)(x-4y)$

**Factorising by pairing**

We can factorise expressions like $2x^2 + xy - 2x - y$ using the method of factorising by pairing:

$2x^2 + xy - 2x - y = x(2x + y) – 1(2x + y)$ (factorise front and back pairs, ensuring both brackets are identical)

$= (2x + y)(x – 1)$

**Exercise B**

Factorise

1) $x^2 - x - 6$

2) $x^2 + 6x - 16$

3) $2x^2 + 5x + 2$

4) $2x^2 - 3x$

5) $3x^2 + 5x - 2$

6) $2y^2 + 17y + 21$

7) $7y^2 - 10y + 3$

8) $10x^2 + 5x - 30$

9) $4x^2 - 25$

10) $x^2 - 3x - xy + 3y^2$

11) $4x^2 - 12x + 8$

12) $16m^2 - 81n^2$

13) $4y^3 - 9a^2 y$

14) $8(x+1)^2 - 2(x+1) - 10$

# Chapter 4: SOLVING QUADRATIC EQUATIONS

A quadratic equation has the form $ax^2 + bx + c = 0$.

There are two methods that are commonly used for solving quadratic equations:
* factorising
* the quadratic formula

Note that not all quadratic equations can be solved by factorising. The quadratic formula can always be used however.

**Method 1: Factorising**

Make sure that the equation is rearranged so that the right hand side is 0. It usually makes it easier if the coefficient of $x^2$ is positive.

**Example 1** : Solve $x^2 - 3x + 2 = 0$

Factorise $(x-1)(x-2) = 0$
Either $(x-1) = 0$ or $(x-2) = 0$
So the solutions are $x = 1$ or $x = 2$

Note: The individual values $x = 1$ and $x = 2$ are called the **roots** of the equation.

**Example 2**: Solve $x^2 - 2x = 0$

Factorise: $x(x-2) = 0$
Either $x = 0$ or $(x-2) = 0$
So $x = 0$ or $x = 2$

**Method 2: Using the formula**

Recall that the roots of the quadratic equation $ax^2 + bx + c = 0$ are given by the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**Example 3**: Solve the equation $2x^2 - 5 = 7 - 3x$

**Solution**: First we rearrange so that the right hand side is 0. We get $2x^2 + 3x - 12 = 0$
We can then tell that $a = 2$, $b = 3$ and $c = -12$.
Substituting these into the quadratic formula gives:

$$x = \frac{-3 \pm \sqrt{3^2 - 4 \times 2 \times (-12)}}{2 \times 2} = \frac{-3 \pm \sqrt{105}}{4}$$ (this is the *surd form* for the solutions)

$$x = -\frac{3}{4} + \frac{1}{4}\sqrt{105} \quad \text{and} \quad x = -\frac{3}{4} - \frac{1}{4}\sqrt{105}$$

**EXERCISE**

1) Use factorisation to solve the following equations:

a) $x^2 + 3x + 2 = 0$

b) $x^2 - 3x - 4 = 0$

c) $x^2 = 15 - 2x$

2) Find the roots of the following equations:

a) $x^2 + 3x = 0$

b) $x^2 - 4x = 0$

c) $4 - x^2 = 0$

3) Solve the following equations either by factorising or by using the formula:

a) $6x^2 - 5x - 4 = 0$

b) $8x^2 - 24x + 10 = 0$

4) Solve the following equations, giving your answers in surd form where appropriate. Some of the equations can't be solved.

a) $x^2 + 7x + 9 = 0$

b) $6 + 3x = 8x^2$

c) $4x^2 - x - 7 = 0$

d) $x^2 - 3x + 18 = 0$

e) $3x^2 + 4x + 4 = 0$

f) $3x^2 = 13x - 16$

# Chapter 5: SIMULTANEOUS EQUATIONS

**Two linear simultaneous equations**

**Example 1**: Solve $3x + 2y = 8$ ① 
$5x + y = 11$ ②

We can solve these equations by eliminating one of the letters from the equations. In the equations above it is simplest to eliminate $y$ by making the coefficients of $y$ the same in both equations. This can be achieved by multiplying equation ② by 2, so that both equations contain $2y$:

$3x + 2y = 8$ ①
$10x + 2y = 22$ $2\times$② = ③

To eliminate the $y$ terms, we subtract equation ③ from equation ①. We get: $7x = 14$
$x = 2$

To find y, we substitute $x = 2$ into one of the original equations. For example if we put it into ②:

$10 + y = 11$
$y = 1$

Therefore, the solution is $x = 2$, $y = 1$.

**Example 2**: Solve $2x + 5y = 16$ ①
$3x - 4y = 1$ ②

**Solution**: We begin by getting the same number of $x$ or $y$ appearing in both equation. We can get $20y$ in both equations if we multiply the top equation by 4 and the bottom equation by 5:

$8x + 20y = 64$ ③
$15x - 20y = 5$ ④

As the SIGNS in front of $20y$ are DIFFERENT, we can eliminate the $y$ terms from the equations by ADDING:

$23x = 69$ ③+④
i.e. $x = 3$

Substituting this into equation ① gives:

$6 + 5y = 16$
$5y = 10$
So… $y = 2$

Therefore, the solution is $x = 3$, $y = 2$.

**Exercise A**: Solve the pairs of simultaneous equations in the following questions:

1) $x + 2y = 7$
$3x + 2y = 9$

2) $x + 3y = 0$
$3x + 2y = -7$

3) $3x - 2y = 4$
$2x + 3y = -6$

4) $9x - 2y = 25$
$4x - 5y = 7$

5) $4a + 3b = 22$
$5a - 4b = 43$

6) $3p + 3q = 15$
$2p + 5q = 14$

**One linear, one non-linear simultaneous equations**

Key is to make $x$ or $y$ the subject of the linear equation and then substitute it into the non-linear equation.

*Example:* Solve $x - 2y = 3$ **I**

$x^2 - 2y^2 - 3y = 5$ **II**

*Solution:* From **I** $x = 2y + 3$

Substitute in **II**

$\Rightarrow \quad (2y + 3)^2 - 2y^2 - 3y = 5$

$\Rightarrow \quad 4y^2 + 12y + 9 - 2y^2 - 3y = 5$

$\Rightarrow \quad 2y^2 + 9y + 4 = 0 \quad \Rightarrow \quad (2y + 1)(y + 4) = 0$

$\Rightarrow \quad y = -½ \text{ or } y = -4$

$\Rightarrow \quad x = 2 \text{ or } x = -5$ using **I** (do not use **II** unless you like doing extra work!)

Check in the quadratic equation

When $x = 2$ and $y = -½$

L.H.S. $= 2^2 - 2(-½)^2 - 3(-½) = 5 =$ R.H.S.

and when $x = -5$ and $y = -4$

L.H.S. $= (-5)^2 - 2(-4)^2 - 3(-4) = 25 - 32 + 12 = 5 =$ R.H.S.

*Answer:* $x = 2,\ y = -½$ or $x = -5,\ y = -4$

**Exercise B**:

Solve each pair of simultaneous equations.

| | | |
|---|---|---|
| **a** $x^2 - y + 3 = 0$<br>$x - y + 5 = 0$ | **b** $2x^2 - y - 8x = 0$<br>$x + y + 3 = 0$ | **c** $x^2 + y^2 = 25$<br>$2x - y = 5$ |
| **d** $x^2 + 2xy + 15 = 0$<br>$2x - y + 10 = 0$ | **e** $x^2 - 2xy - y^2 = 7$<br>$x + y = 1$ | **f** $3x^2 - x - y^2 = 0$<br>$x + y - 1 = 0$ |
| **g** $2x^2 + xy + y^2 = 22$<br>$x + y = 4$ | **h** $x^2 - 4y - y^2 = 0$<br>$x - 2y = 0$ | **i** $x^2 + xy = 4$<br>$3x + 2y = 6$ |

# Chapter 6: LINEAR AND QUADRATIC INEQUALITIES

**Linear inequalities**

Inequalities tell us about the relative size of two values. The variable $x$ for example (which could represent displacement) might be more than 4m in a given situation. Mathematically we could write $x > 4$ and we would read this as "$x$ is greater than 4". This gives us the range of values satisfying the inequality.

Linear Inequalities are solved in a similar manner to linear equations BUT If you are multiplying or dividing the inequality by a negative number you must change the inequality sign round.

Example
Find the set of values of $x$ for which:

$$2x - 1 > 5x - 4$$
$$-1 > 3x - 4$$
$$3 > 3x$$
$$1 > x$$
$$x < 1$$

**Exercise A**

| | | |
|---|---|---|
| **a** $2y - 3 > y + 4$ | **b** $5p + 1 \leq p + 3$ | **c** $x - 2 < 3x - 8$ |
| **d** $a + 11 \geq 15 - a$ | **e** $17 - 2u < 2 + u$ | **f** $5 - b \geq 14 - 3b$ |
| **g** $4x + 23 < x + 5$ | **h** $12 + 3y \geq 2y - 1$ | **i** $16 - 3p \leq 36 + p$ |
| **j** $5(r - 2) > 30$ | **k** $3(1 - 2t) \leq t - 4$ | **l** $2(3 + x) \geq 4(6 - x)$ |

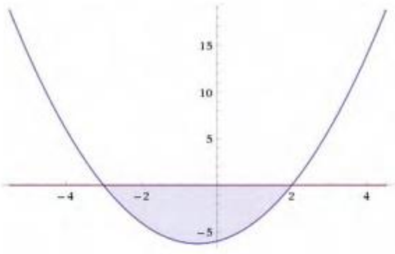
**Quadratic inequalities**

Quadratic inequalities are dealt with in a similar manner to quadratic equations and solved often with aid of a sketch. Some examples are shown below. Remember! The $x$ axis is the line $y = 0$

$x^2 + x - 6 < 0$

$(x - 2)(x + 3) < 0$

The critical values are 2 and -3

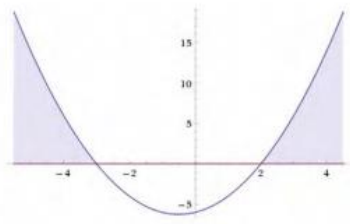



$\therefore -3 < x < 2$

$x^2 + x - 6 > 0$

$(x - 2)(x + 3) > 0$

The critical values are 2 and -3

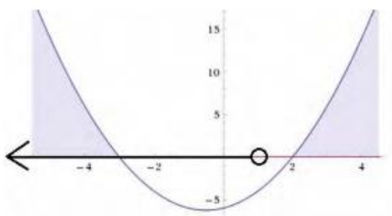



$\therefore x < -3, x > 2$

If we were asked to find the set of values that satisfy BOTH inequalities below we could add the linear inequality to our sketch.

$x^2 + x - 6 > 0$

$2x - 1 > 5x - 4$




We are interest in the set of values where there is both a line and shading. In this case it would be $x < -3$

**Exercise B**

Solve each inequality.

**a** $2x^2 - 9x + 4 \leq 0$

**b** $2r^2 - 5r - 3 < 0$

**c** $2 - p - 3p^2 \geq 0$

**d** $2y^2 + 9y - 5 > 0$

**e** $4m^2 + 13m + 3 < 0$

**f** $9x - 2x^2 \leq 10$

**g** $a^2 + 6 < 8a - 9$

**h** $x(x + 4) \leq 7 - 2x$

**i** $y(y + 9) > 2(y - 5)$

**j** $x(2x + 1) > x^2 + 6$

**k** $u(5 - 6u) < 3 - 4u$

**l** $2t + 3 \geq 3t(t - 2)$

# Chapter 7: COMPLETING THE SQUARE

**When the coefficient of $x^2$ is 1,** take half the coefficient of the term in $x$ into the bracket with an $x$, square the bracket, subtract the squared value away.

Express $x^2 - 6x + 7$ in the form $(x + a)^2 + b$.

The coefficient of $x$ is –6, halve it to give –3 then square to give 9 which is then add subtracted

$$\begin{aligned} x^2 - 6x + 7 &= (x - 3)^2 - (-3)^2 + 7 \\ &= (x - 3)^2 - 9 + 7 \\ &= (x - 3)^2 - 2 \end{aligned}$$

**When the coefficient of $x^2$ is $\neq 1$,** we must factor out the coefficient of $x^2$ first, and then follow the steps above.

Express $-3x^2 - 24x + 5$ in the form $p(x + q)^2 + r$.

The coefficient of $x^2$ is not +1, so we must take out a factor of –3 first.

$$\begin{aligned} -3x^2 - 24x + 5 &= -3(x^2 + 8x) + 5 \\ &= -3[(x + 4)^2 - (4)^2] + 5 \\ &= -3[(x + 4)^2 - 16] + 5 \\ &= -3(x + 4)^2 + 48 + 5 \\ &= -3(x + 4)^2 + 53 \end{aligned}$$

**1** Express in the form $(x + a)^2 + b$

**a** $x^2 + 2x + 4$ **b** $x^2 - 2x + 4$ **c** $x^2 - 4x + 1$ **d** $x^2 + 6x$

**e** $x^2 + 4x + 8$ **f** $x^2 - 8x - 5$ **g** $x^2 + 12x + 30$ **h** $x^2 - 10x + 25$

**i** $x^2 + 6x - 9$ **j** $18 - 4x + x^2$ **k** $x^2 + 3x + 3$ **l** $x^2 + x - 1$

**2** Express in the form $a(x + b)^2 + c$

**a** $2x^2 + 4x + 3$ **b** $2x^2 - 8x - 7$ **c** $3 - 6x + 3x^2$ **d** $4x^2 + 24x + 11$

**e** $-x^2 - 2x - 5$ **f** $1 + 10x - x^2$ **g** $2x^2 + 2x - 1$ **h** $3x^2 - 9x + 5$

**i** $3x^2 - 24x + 48$ **j** $3x^2 - 15x$ **k** $70 + 40x + 5x^2$ **l** $2x^2 + 5x + 2$

# Chapter 8: REARRANGING FORMULAE

We can use algebra to change the subject of a formula. Rearranging a formula is similar to solving an equation – we must do the same to both sides in order to keep the equation balanced.

**Example 1**: Make $x$ the subject of the formula $y = 4x + 3$.

**Solution**: $y = 4x + 3$

Subtract 3 from both sides: $y - 3 = 4x$

Divide both sides by 4; $\dfrac{y-3}{4} = x$

So $x = \dfrac{y-3}{4}$ is the same equation but with $x$ the subject.

**Example 2**: Make $x$ the subject of $y = 2 - 5x$

**Solution**: Notice that in this formula the $x$ term is negative.

$y = 2 - 5x$

Add $5x$ to both sides $y + 5x = 2$ (the $x$ term is now positive)

Subtract $y$ from both sides $5x = 2 - y$

Divide both sides by 5 $x = \dfrac{2-y}{5}$

**Example 3**: The formula $C = \dfrac{5(F-32)}{9}$ is used to convert between ° Fahrenheit and ° Celsius.

We can rearrange to make $F$ the subject.

$$C = \frac{5(F-32)}{9}$$

Multiply by 9 $9C = 5(F-32)$ (this removes the fraction)

Expand the brackets $9C = 5F - 160$

Add 160 to both sides $9C + 160 = 5F$

Divide both sides by 5 $\dfrac{9C+160}{5} = F$

Therefore the required rearrangement is $F = \dfrac{9C+160}{5}$.

**Exercise A**

Make $x$ the subject of each of these formulae:

1) $y = 7x - 1$

2) $y = \dfrac{x+5}{4}$

3) $4y = \dfrac{x}{3} - 2$

4) $y = \dfrac{4(3x-5)}{9}$

**Rearranging equations involving squares and square roots**

**Example 4**: Make $x$ the subject of $x^2 + y^2 = w^2$

**Solution**: $x^2 + y^2 = w^2$

Subtract $y^2$ from both sides: $x^2 = w^2 - y^2$ (this isolates the term involving $x$)

Square root both sides: $x = \pm\sqrt{w^2 - y^2}$

Remember that you can have a positive or a negative square root. We cannot simplify the answer any more.

**Example 5**: Make $a$ the subject of the formula $t = \frac{1}{4}\sqrt{\frac{5a}{h}}$

**Solution**: $t = \frac{1}{4}\sqrt{\frac{5a}{h}}$

Multiply by 4 $4t = \sqrt{\frac{5a}{h}}$

Square both sides $16t^2 = \frac{5a}{h}$

Multiply by $h$: $16t^2 h = 5a$

Divide by 5: $\frac{16t^2 h}{5} = a$

**Exercise B:**

Make $t$ the subject of each of the following

1) $P = \frac{wt}{32r}$

2) $P = \frac{wt^2}{32r}$

3) $V = \frac{1}{3}\pi t^2 h$

4) $P = \sqrt{\frac{2t}{g}}$

5) $Pa = \frac{w(v-t)}{g}$

6) $r = a + bt^2$

**More difficult examples**

Sometimes the variable that we wish to make the subject occurs in more than one place in the formula. In these questions, we collect the terms involving this variable on one side of the equation, and we put the other terms on the opposite side.

**Example 6**: Make $t$ the subject of the formula $a - xt = b + yt$

**Solution**: $a - xt = b + yt$

Start by collecting all the t terms on the right hand side:

Add $xt$ to both sides: $a = b + yt + xt$

Now put the terms without a $t$ on the left hand side:

Subtract $b$ from both sides: $a - b = yt + xt$

Factorise the RHS: $a - b = t(y + x)$

Divide by $(y + x)$: $\dfrac{a-b}{y+x} = t$

So the required equation is $t = \dfrac{a-b}{y+x}$

**Example 7**: Make $W$ the subject of the formula $T - W = \dfrac{Wa}{2b}$

**Solution**: This formula is complicated by the fractional term. We begin by removing the fraction:

Multiply by $2b$: $2bT - 2bW = Wa$

Add $2bW$ to both sides: $2bT = Wa + 2bW$ (this collects the W's together)

Factorise the RHS: $2bT = W(a + 2b)$

Divide both sides by $a + 2b$: $W = \dfrac{2bT}{a+2b}$

**Exercise C**

Make $x$ the subject of these formulae:

1) $ax + 3 = bx + c$

2) $3(x + a) = k(x - 2)$

3) $y = \dfrac{2x+3}{5x-2}$

4) $\dfrac{x}{a} = 1 + \dfrac{x}{b}$

# Chapter 9: INDICES

**Basic rules of indices**

$y^4$ means $y \times y \times y \times y$.

4 is called the **index** (plural: indices), **power** or **exponent** of *y*.

There are 3 basic rules of indices:

| | | | |
|---|---|---|---|
| 1) | $a^m \times a^n = a^{m+n}$ | e.g. | $3^4 \times 3^5 = 3^9$ |
| 2) | $a^m \div a^n = a^{m-n}$ | e.g. | $3^8 \times 3^6 = 3^2$ |
| 3) | $(a^m)^n = a^{mn}$ | e.g. | $\left(3^2\right)^5 = 3^{10}$ |

**Further examples**

$y^4 \times 5y^3 = 5y^7$

$4a^3 \times 6a^2 = 24a^5$ (multiply the numbers and multiply the *a*'s)

$2c^2 \times \left(-3c^6\right) = -6c^8$ (multiply the numbers and multiply the *c*'s)

$24d^7 \div 3d^2 = \dfrac{24d^7}{3d^2} = 8d^5$ (divide the numbers and divide the *d* terms i.e. by subtracting the powers)

**Exercise A**

Simplify the following:

1) $b \times 5b^5 =$ (Remember that $b = b^1$)

2) $3c^2 \times 2c^5 =$

3) $b^2c \times bc^3 =$

4) $2n^6 \times (-6n^2) =$

5) $8n^8 \div 2n^3 =$

6) $d^{11} \div d^9 =$

7) $\left(a^3\right)^2 =$

8) $\left(-d^4\right)^3 =$

**More complex powers**

**Zero index:**
Recall from GCSE that

$$a^0 = 1.$$

This result is true for any non-zero number *a*.

Therefore $\quad 5^0 = 1 \qquad \left(\frac{3}{4}\right)^0 = 1 \qquad (-5.2304)^0 = 1$

**Negative powers**
A power of -1 corresponds to the reciprocal of a number, i.e. $a^{-1} = \frac{1}{a}$

Therefore $\quad 5^{-1} = \frac{1}{5}$

$$0.25^{-1} = \frac{1}{0.25} = 4$$

$$\left(\frac{4}{5}\right)^{-1} = \frac{5}{4}$$

(you find the reciprocal of a fraction by swapping the numerator and denominator over)

This result can be extended to more general negative powers: $a^{-n} = \frac{1}{a^n}$.

This means:

$$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

$$2^{-4} = \frac{1}{2^4} = \frac{1}{16}$$

$$\left(\frac{1}{4}\right)^{-2} = \left(\left(\frac{1}{4}\right)^{-1}\right)^2 = \left(\frac{4}{1}\right)^2 = 16$$

**Fractional powers:**
Fractional powers correspond to roots:

$$a^{1/2} = \sqrt{a} \qquad a^{1/3} = \sqrt[3]{a} \qquad a^{1/4} = \sqrt[4]{a}$$

In general:

$$a^{1/n} = \sqrt[n]{a}$$

Therefore:

$$8^{1/3} = \sqrt[3]{8} = 2 \qquad 25^{1/2} = \sqrt{25} = 5 \qquad 10000^{1/4} = \sqrt[4]{10000} = 10$$

**Exercise B:**

Find the value of:

1) $4^{1/2}$

2) $27^{1/3}$

3) $\left(\frac{1}{9}\right)^{1/2}$

4) $5^{-2}$

5) $18^{0}$

6) $7^{-1}$

7) $\left(\frac{2}{3}\right)^{-2}$

8) $(0.04)^{1/2}$

Simplify each of the following:

9) $2a^{1/2} \times 3a^{5/2}$

10) $x^{3} \times x^{-2}$

11) $\left(x^{2} y^{4}\right)^{1/2}$

**Exercise C**:

Solve each equation.

| | | | |
|---|---|---|---|
| **a** $x^{\frac{1}{2}} = 6$ | **b** $x^{\frac{1}{3}} = 5$ | **c** $x^{-\frac{1}{2}} = 2$ | **d** $x^{-\frac{1}{4}} = \frac{1}{3}$ |
| **e** $x^{\frac{3}{2}} = 8$ | **f** $x^{\frac{2}{3}} = 16$ | **g** $x^{\frac{4}{3}} = 81$ | **h** $x^{-\frac{3}{2}} = 27$ |

# Chapter 10: SURDS

| |
|---|
| Surds are irrational numbers and are said to be 'exact values'. Don't be tempted to try and write a truncated or rounded decimal answer. Leave your answer as a surd. Calculations in this form will be more accurate & easy to perform.<br>Generally you will either have to simplify surds, carry out basic calculations with the 4 operations or rationalise the denominator of a fraction with a surd in. |
| ***Some Basic Surd Laws*** |
| $\sqrt{a} \times \sqrt{a} = a$<br>$a\sqrt{c} \times b\sqrt{d} = ab\sqrt{cd}$<br>$\sqrt{a} + \sqrt{a} = 2\sqrt{a}$<br>Just try these with numeric values. You will see many break down into integer values |
| ***Simplifying Surds*** |
| Breaking surds down (simplifying) is just a case of prime factorising e.g. $\sqrt{8} = \sqrt{2} \times \sqrt{2} \times \sqrt{2} = 2\sqrt{2}$ |
| ***Adding and Subtracting Surds*** |
| You can only add and subtract 'like surds' BUT some will simplify to allow you to do that e.g.<br>$\sqrt{3} + 4\sqrt{3} = 5\sqrt{3}$ or $7\sqrt{2} - 3\sqrt{2} = 4\sqrt{2}$ are examples of surd adding/subtracting without prior simplification.<br>Sometimes you will have to simplify first<br>e.g. $\sqrt{50} + \sqrt{8} = (\sqrt{5} \times \sqrt{5} \times \sqrt{2}) + (\sqrt{2} \times \sqrt{2} \times \sqrt{2}) = 5\sqrt{2} + 2\sqrt{2} = 7\sqrt{2}$ |

**Exercise A**

Simplify

(a) $\sqrt{12}$ (b) $\sqrt{44}$ (c) $\sqrt{45}$ (d) $\sqrt{50}$

(e) $\sqrt{75}$ (f) $\sqrt{24}$ (g) $\sqrt{63}$ (h) $\sqrt{200}$

| |
|---|
| ***Expanding Brackets*** |
| You may be expected to expand single or double brackets. Most of these questions require full workings e.g.<br>*Write* $(2+\sqrt{3})(5-\sqrt{27})$ *in the form* $a + b\sqrt{3}$<br>$(2+\sqrt{3})(5-\sqrt{27}) = (2+\sqrt{3})(5-3\sqrt{3}) = (2)(5) + (2)(-3\sqrt{3}) + (\sqrt{3})(5) + (\sqrt{3})(-3\sqrt{3}) = 10 - 6\sqrt{3} + 5\sqrt{3} - 27 = -17 - \sqrt{3}$ |

**Exercise B**

Expand and simplify

(a) $(\sqrt{3} + 2)(\sqrt{3} + 3)$ (b) $(\sqrt{5} - 1)(\sqrt{5} + 2)$

(c) $(\sqrt{3} - 4)(2 + \sqrt{5})$ (d) $(\sqrt{2} + \sqrt{3})(\sqrt{3} + \sqrt{5})$

(e) $(\sqrt{5} - \sqrt{2})(\sqrt{3} + \sqrt{2})$ (f) $(\sqrt{7} + \sqrt{3})(\sqrt{5} + \sqrt{2})$

## Rationalising the denominator

Rationalising the denominator means simplifying the fraction so there is a rational number in the denominator not a surd. You do this by multiplying the numerator and denominator by a surd of the opposite sign.

Rationalise the denominator of $\frac{5}{3-\sqrt{2}}$.

Rationalise the denominator by multiplying the numerator and denominator by $3+\sqrt{2}$.

$$\frac{5}{3-\sqrt{2}} = \frac{5(3+\sqrt{2})}{(3-\sqrt{2})(3+\sqrt{2})} = \frac{15+5\sqrt{2}}{9+3\sqrt{2}-3\sqrt{2}-2} = \frac{15+5\sqrt{2}}{7}$$

**Exercise C**

Rationalise the denominator giving your answers in the form $a + b\sqrt{c}$ where a, b and c are rational numbers.

a $\frac{1}{1+\sqrt{2}}$  b $\frac{1}{5-\sqrt{3}}$  c $\frac{7}{4-\sqrt{5}}$

d $\frac{4}{1+\sqrt{6}}$  e $\frac{\sqrt{5}}{1-\sqrt{5}}$  f $\frac{6+\sqrt{2}}{8-\sqrt{2}}$

# CHAPTER 11 – ANSWERS

## CHAPTER 1 – EXPANDING BRACKETS

<u>Ex A</u>

1) $28x + 35$ 2) $-15x + 21$ 3) $-7a + 4$ 4) $6y + 3y^2$ 5) $-4x – 4$
6) $7x – 1$ 7) $x^2 + 5x + 6$ 8) $t^2 – 7t + 10$ 9) $6x^2 + xy – 12y^2$
10) $4x^2 + 4x – 24$ 11) $4y^2 – 1$ 12) $12 + 17x – 5x^2$

<u>Ex B</u>

1) $x^2 – 2x + 1$ 2) $9x^2 + 30x + 25$ 3) $49x^2 – 28x + 4$ 4) $x^2 – 4$ 5) $9x^2 -1$
6) $25y^2 – 9$

## CHAPTER 2 – SOLVING LINEAR EQUATIONS

<u>Ex A</u> 1) 7 2) 3 3) 1½ 4) 2 5) -3/5 6) -7/3

<u>Ex B</u> 1) 2.4 2) 5 3) 1 4) ½

<u>Ex C</u> 1) 7 2) 15 3) 24/7 4) 35/3 5) 3 6) 2 7) 9/5 8) 5

<u>Ex D</u> 1) 34, 36, 38 2) 9.875, 29.625 3) 24, 48

## CHAPTER 3 – FACTORISING

<u>Ex A</u>

1) $x(3 + y)$ 2) $2x(2x – y)$ 3) $pq(q – p)$ 4) $3q(p – 3q)$ 5) $2x^2(x - 3)$ 6) $4a^3b^2(2a^2 – 3b^2)$
7) $(y – 1)(5y + 3)$

<u>Ex B</u>

1) $(x – 3)(x + 2)$ 2) $(x + 8)(x – 2)$ 3) $(2x + 1)(x + 2)$ 4) $x(2x – 3)$ 5) $(3x -1 )(x + 2)$
6) $(2y + 3)(y + 7)$ 7) $(7y – 3)(y – 1)$ 8) $5(2x – 3)(x + 2)$ 9) $(2x + 5)(2x – 5)$ 10) $(x – 3)(x – y)$
11) $4(x – 2)(x – 1)$ 12) $(4m – 9n)(4m + 9n)$ 13) $y(2y – 3a)(2y + 3a)$ 14) $2(4x + 5)(x – 4)$

## CHAPTER 4 – SOLVING QUADRATIC EQUATIONS

1) a) -1, -2 b) -1, 4 c) -5, 3

2) a) 0, -3 b) 0, 4 c) 2, -2

3) a) -1/2, 4/3 b) 0.5, 2.5

4) a) $x = \frac{-7\pm\sqrt{13}}{2}$

b) $x = \frac{3\pm\sqrt{201}}{16}$

c) $x = \frac{1\pm\sqrt{113}}{8}$

d) no solutions e) no solutions f) no solutions

## CHAPTER 5 – SIMULTANEOUS EQUATIONS

Ex A

1) $x = 1, y = 3$ 2) $x = -3, y = 1$ 3) $x = 0, y = -2$ 4) $x = 3, y = 1$

5) $a = 7,\ b = -2$ 6) $p = 11/3,\ q = 4/3$

Ex B

**a** subtracting
$x^2 - x - 2 = 0$
$(x + 1)(x - 2) = 0$
$x = -1$ or $2$
$\therefore\ x = -1, y = 4$
or $x = 2, y = 7$

**b** adding
$2x^2 - 7x + 3 = 0$
$(2x - 1)(x - 3) = 0$
$x = \frac{1}{2}$ or $3$
$\therefore\ x = \frac{1}{2}, y = -\frac{7}{2}$
or $x = 3, y = -6$

**c** $y = 2x - 5$
sub
$x^2 + (2x - 5)^2 = 25$
$x^2 - 4x = 0$
$x(x - 4) = 0$
$x = 0$ or $4$
$\therefore\ x = 0, y = -5$
or $x = 4, y = 3$

**d** $y = 2x + 10$
sub.
$x^2 + 2x(2x + 10) + 15 = 0$
$x^2 + 4x + 3 = 0$
$(x + 3)(x + 1) = 0$
$x = -3$ or $-1$
$\therefore\ x = -3, y = 4$
or $x = -1, y = 8$

**e** $y = 1 - x$
sub.
$x^2 - 2x(1 - x) - (1 - x)^2 = 7$
$x^2 = 4$
$x = \pm 2$
$\therefore\ x = -2, y = 3$
or $x = 2, y = -1$

**f** $y = 1 - x$
sub.
$3x^2 - x - (1 - x)^2 = 0$
$2x^2 + x - 1 = 0$
$(2x - 1)(x + 1) = 0$
$x = -1$ or $\frac{1}{2}$
$\therefore\ x = -1, y = 2$
or $x = \frac{1}{2}, y = \frac{1}{2}$

**g** $y = 4 - x$
sub.
$2x^2 + x(4 - x) + (4 - x)^2 = 22$
$x^2 - 2x - 3 = 0$
$(x + 1)(x - 3) = 0$
$x = -1$ or $3$
$\therefore\ x = -1, y = 5$
or $x = 3, y = 1$

**h** $x = 2y$
sub.
$(2y)^2 - 4y - y^2 = 0$
$3y^2 - 4y = 0$
$y(3y - 4) = 0$
$y = 0$ or $\frac{4}{3}$
$\therefore\ x = 0, y = 0$
or $x = \frac{8}{3}, y = \frac{4}{3}$

**i** $y = 3 - \frac{3}{2}x$
sub.
$x^2 + x(3 - \frac{3}{2}x) = 4$
$x^2 - 6x + 8 = 0$
$(x - 2)(x - 4) = 0$
$x = 2$ or $4$
$\therefore\ x = 2, y = 0$
or $x = 4, y = -3$

## CHAPTER 6 – LINEAR AND QUADRATIC INEQUALITIES

Ex A

**a** $y > 7$

**b** $4p \leq 2$
$p \leq \frac{1}{2}$

**c** $6 < 2x$
$x > 3$

**d** $2a \geq 4$
$a \geq 2$

**e** $15 < 3u$
$u > 5$

**f** $2b \geq 9$
$b \geq \frac{9}{2}$

**g** $3x < -18$
$x < -6$

**h** $y \geq -13$

**i** $-20 \leq 4p$
$p \geq -5$

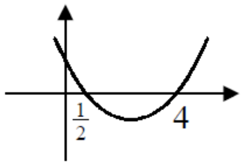
**j** $r - 2 > 6$
$r > 8$

**k** $3 - 6t \leq t - 4$
$7 \leq 7t$
$t \geq 1$

**l** $6 + 2x \geq 24 - 4x$
$6x \geq 18$
$x \geq 3$

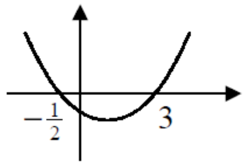
Ex B

**a** $(2x-1)(x-4) \le 0$




$\therefore\ \frac{1}{2} \le x \le 4$

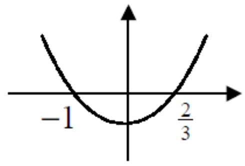
**b** $(2r+1)(r-3) < 0$




$\therefore\ -\frac{1}{2} < r < 3$

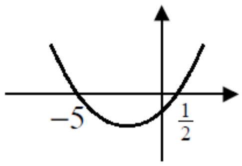
**c** $3p^2 + p - 2 \le 0$

$(3p-2)(p+1) \le 0$

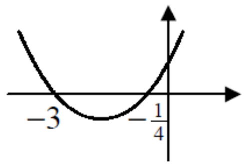



$\therefore\ -1 \le p \le \frac{2}{3}$

**d** $(2y-1)(y+5) > 0$

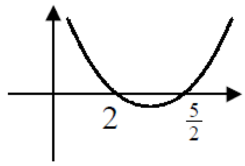



**e** $(4m+1)(m+3) < 0$



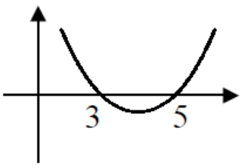

**f** $2x^2 - 9x + 10 \ge 0$

$(2x-5)(x-2) \ge 0$




**g** $a^2 - 8a + 15 < 0$

$(a-3)(a-5) < 0$

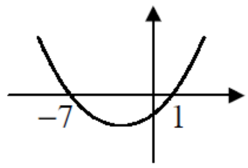



$\therefore\ 3 < a < 5$

**h** $x^2 + 4x \le 7 - 2x$

$x^2 + 6x - 7 \le 0$

$(x+7)(x-1) \le 0$

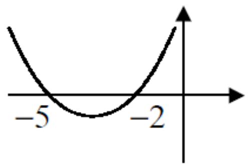



$\therefore\ -7 \le x \le 1$

**i** $y^2 + 9y > 2y - 10$

$y^2 + 7y + 10 > 0$

$(y+5)(y+2) > 0$

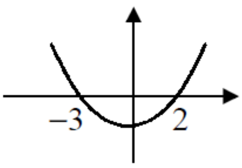



$\therefore\ y < -5$ or $y > -2$

**j** $2x^2 + x > x^2 + 6$

$x^2 + x - 6 > 0$

$(x+3)(x-2) < 0$

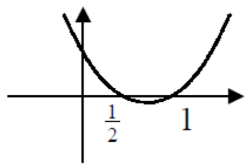



$\therefore\ -3 < x < -2$

**k** $5u - 6u^2 < 3 - 4u$

$2u^2 - 3u + 1 > 0$

$(2u-1)(u-1) > 0$

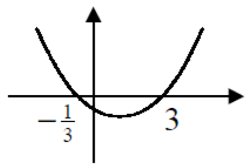



$\therefore\ u < \frac{1}{2}$ or $u > 1$

**l** $2t + 3 \ge 3t^2 - 6t$

$3t^2 - 8t - 3 \le 0$

$(3t+1)(t-3) \le 0$




$\therefore\ -\frac{1}{3} \le t \le 3$

## CHAPTER 7 – COMPLETING THE SQUARE

**1** **a** $= (x+1)^2 - 1 + 4$
$= (x+1)^2 + 3$

**b** $= (x-1)^2 - 1 + 4$
$= (x-1)^2 + 3$

**c** $= (x-2)^2 - 4 + 1$
$= (x-2)^2 - 3$

**d** $= (x+3)^2 - 9$

**e** $= (x+2)^2 - 4 + 8$
$= (x+2)^2 + 4$

**f** $= (x-4)^2 - 16 - 5$
$= (x-4)^2 - 21$

**g** $= (x+6)^2 - 36 + 30$
$= (x+6)^2 - 6$

**h** $= (x-5)^2 - 25 + 25$
$= (x-5)^2$

**i** $= (x+3)^2 - 9 - 9$
$= (x+3)^2 - 18$

**j** $= (x-2)^2 - 4 + 18$
$= (x-2)^2 + 14$

**k** $= (x+\frac{3}{2})^2 - \frac{9}{4} + 3$
$= (x+\frac{3}{2})^2 + \frac{3}{4}$

**l** $= (x+\frac{1}{2})^2 - \frac{1}{4} - 1$
$= (x+\frac{1}{2})^2 - \frac{5}{4}$

**a** $= 2[x^2 + 2x] + 3$
$= 2[(x+1)^2 - 1] + 3$
$= 2(x+1)^2 + 1$

**b** $= 2[x^2 - 4x] - 7$
$= 2[(x-2)^2 - 4] - 7$
$= 2(x-2)^2 - 15$

**c** $= 3[x^2 - 2x] + 3$
$= 3[(x-1)^2 - 1] + 3$
$= 3(x-1)^2$

**d** $= 4[x^2 + 6x] + 11$
$= 4[(x+3)^2 - 9] + 11$
$= 4(x+3)^2 - 25$

**e** $= -[x^2 + 2x] - 5$
$= -[(x+1)^2 - 1] - 5$
$= -(x+1)^2 - 4$

**f** $= -[x^2 - 10x] + 1$
$= -[(x-5)^2 - 25] + 1$
$= -(x-5)^2 + 26$

**g** $= 2[x^2 + x] - 1$
$= 2[(x+\frac{1}{2})^2 - \frac{1}{4}] - 1$
$= 2(x+\frac{1}{2})^2 - \frac{3}{2}$

**h** $= 3[x^2 - 3x] + 5$
$= 3[(x-\frac{3}{2})^2 - \frac{9}{4}] + 5$
$= 3(x-\frac{3}{2})^2 - \frac{7}{4}$

**i** $= 3[x^2 - 8x] + 48$
$= 3[(x-4)^2 - 16] + 48$
$= 3(x-4)^2$

**j** $= 3[x^2 - 5x]$
$= 3[(x-\frac{5}{2})^2 - \frac{25}{4}]$
$= 3(x-\frac{5}{2})^2 - \frac{75}{4}$

**k** $= 5[x^2 + 8x] + 70$
$= 5[(x+4)^2 - 16] + 70$
$= 5(x+4)^2 - 10$

**l** $= 2[x^2 + \frac{5}{2}x] + 2$
$= 2[(x+\frac{5}{4})^2 - \frac{25}{16}] + 2$
$= 2(x+\frac{5}{4})^2 - \frac{9}{8}$

## CHAPTER 8 – REARRANGING FORMULAE

Ex A

1) $x = \dfrac{y+1}{7}$ 2) $x = 4y - 5$ 3) $x = 3(4y+2)$ 4) $x = \dfrac{9y+20}{12}$

Ex B

1) $t = \dfrac{32rP}{w}$ 2) $t = \pm\sqrt{\dfrac{32rP}{w}}$ 3) $t = \pm\sqrt{\dfrac{3V}{\pi h}}$ 4) $t = \dfrac{P^2 g}{2}$ 5) $t = v - \dfrac{Pag}{w}$ 6) $t = \pm\sqrt{\dfrac{r-a}{b}}$

Ex C

1) $x = \dfrac{c-3}{a-b}$ 2) $x = \dfrac{3a+2k}{k-3}$ 3) $x = \dfrac{2y+3}{5y-2}$ 4) $x = \dfrac{ab}{b-a}$

## CHAPTER 9 - INDICES

**Ex A**

1) $5b^6$ 2) $6c^7$ 3) $b^3c^4$ 4) $-12n^8$ 5) $4n^5$ 6) $d^2$ 7) $a^6$ 8) $-d^{12}$

**Ex B**

1) 2 2) 3 3) 1/3 4) 1/25 5) 1 6) 1/7 7) 9/4 8) 0.2 9) $6a^3$ 10) $x$ 11) $xy^2$

**Ex C**

**a** $x = 6^2 = 36$

**b** $x = 5^3 = 125$

**c** $x^{\frac{1}{2}} = \frac{1}{2}$
$x = (\frac{1}{2})^2 = \frac{1}{4}$

**d** $x^{\frac{1}{4}} = 3$
$x = 3^4 = 81$

**e** $x^{\frac{1}{2}} = \sqrt[3]{8} = 2$
$x = 2^2 = 4$

**f** $x^{\frac{1}{3}} = \pm\sqrt{16} = \pm 4$
$x = (\pm 4)^3 = \pm 64$

**g** $x^{\frac{1}{3}} = \pm\sqrt[4]{81} = \pm 3$
$x = (\pm 3)^3 = \pm 27$

**h** $x^{\frac{3}{2}} = \frac{1}{27}$
$x^{\frac{1}{2}} = \sqrt[3]{\frac{1}{27}} = \frac{1}{3}$
$x = (\frac{1}{3})^2 = \frac{1}{9}$

## CHAPTER 10 – SURDS

### Ex A: Simplifying surds

| | | | |
|---|---|---|---|
| (a) $2\sqrt{3}$ | (b) $2\sqrt{11}$ | (c) $3\sqrt{5}$ | (d) $5\sqrt{2}$ |
| (e) $5\sqrt{3}$ | (f) $2\sqrt{6}$ | (g) $3\sqrt{7}$ | (h) $10\sqrt{2}$ |

### Ex B: Expanding brackets

(a) $9 + 5\sqrt{3}$ (b) $3 + \sqrt{5}$

(c) $-8 + 2\sqrt{3} - 4\sqrt{5} + \sqrt{15}$ (d) $3 + \sqrt{6} + \sqrt{10} + \sqrt{15}$

(e) $-2 - \sqrt{6} + \sqrt{10} + \sqrt{15}$ (f) $\sqrt{6} + \sqrt{14} + \sqrt{15} + \sqrt{35}$

### Ex C: Rationalising the denominator

a $\dfrac{(1-\sqrt{2})}{-1}$ b $\dfrac{5+\sqrt{3}}{22}$ c $\dfrac{7(4+\sqrt{5})}{11}$ d $\dfrac{4(1-\sqrt{6})}{-5}$

e $\dfrac{5+\sqrt{5}}{-4}$ f $\dfrac{25+7\sqrt{2}}{31}$